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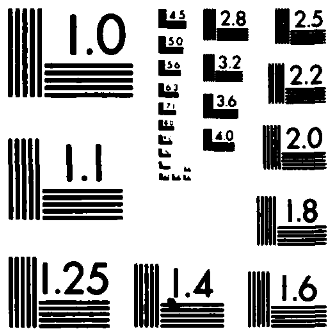

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Mathematical analyses of motion perception have established minimum combinations of points and distinct views that are sufficient to recover three-dimensional (3D) structure from two-dimensional (2D) images, using such regularities as rigid motion, fixed axis of rotation, and constant angular velocity. To determine whether human subjects could recover 3D information at these theoretical levels, we presented subjects with pairs of displays and asked them to determine whether they

20. (continued)

represented the same or different 3D structures. Number of points was varied between 2 and 5; number of views was varied between 2 and 6; and the motion was either fixed axis with constant angular velocity, fixed axis with variable velocity, or variable axis with variable velocity. Accuracy increased with views, but decreased with points, apparently due to the increased difficulty of the comparison task as the structure became more complex. Subjects' performance exceeded theoretical expectations, implying that they exploited regularities in addition to those in the theoretical analyses. Some possible additional regularities, and possible grouping effects, are discussed.

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Abstract

Mathematical analyses of motion perception have established minimum combinations of points and distinct views that are sufficient to recover three-dimensional (3D) structure from two-dimensional (2D) images, using such regularities as rigid motion, fixed axis of rotation, and constant angular velocity. To determine whether human subjects could recover 3D information at these theoretical levels, we presented subjects with pairs of displays and asked them to determine whether they represented the same or different 3D structures. Number of points was varied between 2 and 5; number of views was varied between 2 and 6; and the motion was either fixed axis with constant angular velocity, fixed axis with variable velocity, or variable axis with variable velocity. Accuracy increased with views, but decreased with points, apparently due to the increased difficulty of the comparison task as the structure became more complex. Subjects' performance exceeded theoretical expectations, implying that they exploited regularities in addition to those in the theoretical analyses. Some possible additional regularities, and possible grouping effects, are discussed.

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1. Introduction

Theoretical investigations of visual motion have provided a number of specific analyses of the minimum number of points and views required to recover three-dimensional (3D) structure from two-dimensional (2D) images. Recovery of 3D structure, in this context, is defined as determining the x, y, and z coordinate of each point, up to a scale factor. These analyses differ in the constraints that are imposed. Ullman (1979) showed that under a rigidity constraint, three views of four noncoplanar points are sufficient to recover structure in an orthographic projection, up to a reflection about the frontal plane. The required numbers of points and views are reduced by adding further constraints, such as planarity (Hoffman & Flinchbaugh, 1982), fixed axis of rotation (Hoffman & Bennett, 1986; Webb & Aggarwal, 1981), and constant angular velocity (Hoffman & Bennett, 1985). These proofs are summarized in Table 1.

Table 1

Sufficient Conditions for the Recovery of 3D Structure

Number of Distinct Views	Number of Points: 2	Number of Points: 3	Number of Points: 4
2		Pairwise-rigid and planar motion [a]	
3	Rigid planar motion [a] Rigid fixed axis motion parallel to image plane, constant angular velocity [b]	Rigid fixed axis motion [b]	Rigid motion [c] Nonrigid fixed axis motion [d]
4	Nonrigid fixed axis motion [d] Rigid fixed axis motion, constant angular velocity [e]		
5	Rigid fixed axis motion [e]		

[a] Hoffman & Flinchbaugh, 1982; [b] Hoffman & Bennett, 1986; [c] Ullman, 1979; [d] Bennett & Hoffman, 1985; [e] Hoffman & Bennett, 1985

A number of empirical studies have addressed issues related to theoretical analyses of the recovery of structure from motion. Several studies (for example, Braunstein & Andersen, 1985; Schwartz & Sperling, 1983; Todd, 1985) have questioned the generality of the rigidity constraint. Other studies have considered the recovery of structure with small numbers of views or with small numbers of points. Lappin, Doner, & Kottas (1980) found that subjects could make accurate judgments based on 3D structure with two perspective views of 512 points. Lappin and Fuqua (1983) found a high level of accuracy for relative depth judgments with 120° rotations of three-point configurations. There have not been studies, however, of the recovery of structure from motion using the minimum combinations of points and views found in the theoretical analyses discussed above.

There are several reasons why these theoretical analyses should be considered empirically. First, it is worthwhile to determine whether the performance of human observers approaches the performance of the "ideal" observers in these analyses. Can the human observer recover 3D structure at the minimum combinations of points and views? Second, it is useful to know whether performance improves as predicted by these theoretical analyses when constraints, in addition to rigidity, are imposed on the displays. Specifically, can structure be recovered with fewer points and views when the axis of rotation is fixed and when a constant angular velocity is maintained across views? Third, empirical studies may suggest other constraints used by human observers that have not been considered in theoretical analyses.

On the negative side, one can question the ecological validity of minimum information displays, and of orthographic projections in particular. These displays are clearly special cases. Visual perception normally occurs in richly textured environments with continuous observation. Orthographic projection simulates an infinite viewing distance, eliminating the perspective effects found in normal vision. With these considerations in mind, we still believe that these displays provide a useful starting point for bringing together specific mathematical analyses with psychophysical procedures.

There are at least two fundamental difficulties in applying a psychophysical approach to the testing of theoretical analyses of the recovery of structure from motion. The first stems from the definition given above, according to which recovery of structure consists of determining coordinates in 3D space. This definition provides a suitable measure for computer simulations, but it is not reasonable to expect a human subject to call out coordinates while observing a group of points undergoing a rotation. Some dependent variable is needed, one that is logically related to the recovery of structure but that is based on a reasonable human response. This will be discussed further in the following paragraphs.

The second difficulty is inherent in the task--recovering 3D structure from 2D images. The information for the recovery of the structure must be available in the images, and therefore any task given the subject to determine whether the structure has been recovered must be possible on the basis of the images. How, then, do we know that the subject is not performing the task on the basis of some 2D characteristic of the images without recovering the 3D structure? There is no way, in principle, to be certain of this. As discussed below, the best we can do is to try to find and eliminate any 2D regularities that a subject could use to perform the task without also recovering the 3D structure.

In the first task that we used in pilot studies, displays were generated consisting of three, four or eight points that were the vertices of regular polygons. Each display was paired with a polygon in which the location of one of the vertices relative to the others was altered by a controlled amount, so that the polygon was no longer regular. The subjects viewed the displays side by side. To prevent direct image comparisons, the two polygons were never displayed in the same orientation. Several experienced subjects reported noticeable regularities in the 2D images of the regular polygons, regularities that made it possible for them to distinguish between the regular and irregular polygons in each pair. Although all of these apparent regularities could not be described precisely by the subjects, they appeared to be related to the use of regular polygons as the standard stimuli, and we therefore abandoned that approach.

Instead, we used displays consisting of sets of points that were randomly generated (under restrictions described in the Method section). For each display, a comparison display was presented that was identical to the standard or had one point moved to a different position. The subject's task was to indicate whether the 3D structures represented by the two displays were the same or different. The rationale for using a comparison task as a measure of recovery of structure is that subjects must recover the 3D structure in order to determine whether the displays represent the same or different configurations. Although the task can be performed by comparing substructures if the number of points exceeds the minimum required, it should be necessary to use all of the points to recover the 3D structure of a configuration at the minimum levels.

For the reasons stated above, the two displays were presented out of phase. This probably required the subject to mentally rotate one or both structures to compare them. This extra step of mental rotation, between recovery of the structures and the behavioral response, could have prevented subjects from responding accurately (above chance) to the combinations of small numbers of points and small numbers of views listed in Table 1. As we will indicate below, this did not seem to be the case in our experiment.

Each of the mathematical analyses in Table 1 gives sufficiency conditions for recovering the third dimension if one assumes some specific regularity or regularities in the motion of the simulated object. The individual analyses do not make predictions about improvements in performance with increasing numbers of points or views, or with further constraints. If all of these analyses were instantiated in the visual system, however, we would expect the following results for the numbers of points and views and the motion constraints included in the present experiment:

(1) Accuracy should increase with the number of distinct views.

(2) Accuracy should increase with the number of points.

(3) Accuracy should increase with increasing constraints, from variable axis to fixed axis to fixed axis with constant angular velocity.

In addition to these general trends, accuracy should increase from chance to above chance at the critical combinations of points, views, and constraints listed in Table 1: (a) four points, three views, no added constraints (rigidity only); (b) three points, three views, rigidity and fixed axis constraints; (c) two points, five views, rigidity and fixed axis constraints; and (d) two points, four views, rigidity, fixed axis, and constant angular velocity.

2. Method

2.1 *Subjects*

The subjects were five graduate students and one undergraduate from the School of Social Sciences at the University of California, Irvine. Subjects were paid for their participation. One subject (No. 1), had been directly involved in the research, two were slightly acquainted with the research (Nos. 2 and 3), and three were naive. Acuity of at least 20/40 (Snellen eye chart) was required in the eye the subject used throughout the course of the experiment. Each subject met a performance criterion of 75% or better (overall correct on the *same/different* judgments) during a screening session consisting of 200 trials with 30 views, 3-5 points, and fixed axis/constant angular velocity. Two of the subjects (Nos. 5 and 6), both naive, were run without feedback. The remaining subjects were run with feedback on all trials.

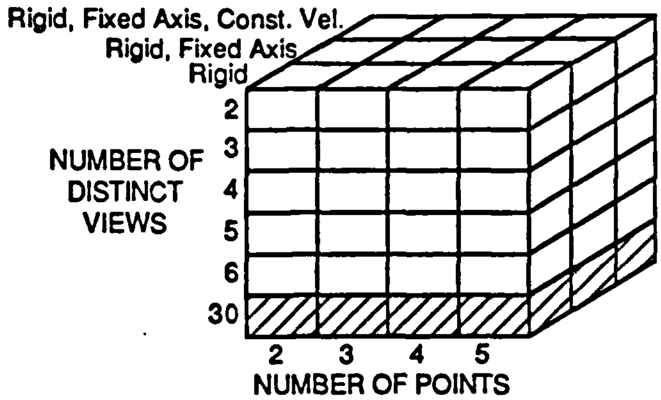

Figure 1. Experimental design.

2.2 *Design*

We examined three independent variables: number of points in a simulated object, number of distinct views presented, and motion constraints (see Figure 1). The number of points in a simulated object ranged from two to five. The number of distinct views presented ranged from two to six. The motion conditions examined were: (a) fixed axis of rotation with constant angular velocity, (b) fixed axis of rotation with a variable angular velocity, and (c) variable axis of rotation. All of the independent variables were run within subjects. Each subject responded to 50 trials of each of the 60 stimulus conditions. Each subject also responded to 75 30-view "baseline" trials at each combination of number of points and motion condition.

2.3 *Stimuli*

A stimulus consisted of two to five light green dots, changing in position, against a dark green background. Each stimulus simulated points on a rigid object rotating in depth. Preliminary point positions for an object were selected at random (without replacement) from a uniform distribution of 225 potential point positions on the surface of a unit-radius sphere. To avoid any unintended regularities in the projection that might have resulted from all points being equidistant from the center of rotation, the distance of each point to the center of the sphere was randomly perturbed within a range of ±0.2 units. This configuration of points was defined as the standard object. For *same* trials, the comparison object was identical to the standard object. For *different* trials, the following method was used to generate the comparison object: One of the points on the standard object was moved to one of the 225 potential point positions that was unoccupied. The point to be moved and the new position were selected at random. If the root mean square (RMS) of the changes in distance (standard object distance minus comparison object distance) from the moved point to all other points in the simulated object did not exceed 0.7 units, these simulated objects were discarded and a new standard was generated. The minimum RMS distance criterion was determined, through pilot studies, to provide a better than 0.8 overall proportion correct for an experienced subject in fixed axis/constant angular velocity displays. As a result of this criterion, and the restrictions described below, the RMS difference varied between 0.7 and 1.88, with a mean of 0.90 and a standard deviation of 0.30.

In order to avoid the possibility of subjects making direct comparisons of the 2D projections, the two simulated objects were set at different initial orientations: 20° slant, 0° tilt for the standard and 50° slant, 0° tilt for the comparison object. (See Stevens, 1983, for a discussion of slant and tilt.) In addition, the standard and comparison objects were always out of phase, with their initial phase difference randomly varied within a range of 40° to 140°.

Each stimulus display consisted of a sequence of distinct, orthogonal views of a simulated object undergoing a specific type of motion in three dimensions. In order to allow subjects sufficient time to observe the displays and make a judgment, the sequence of views was oscillated (e.g., 1, 2, ... n-1, n, n-1, ... 2, 1, 2, ...) at a rate of 16 views per sec until the subject responded. (If the subject did not respond within 60 sec, the trial was repeated at the end of the session.) For the fixed axis conditions, these views were rotations from the initial orientation about an axis at 20° slant, 0° tilt for the standard and 50° slant, 0° tilt for the comparison object. For constant angular velocity conditions, the rotation between successive views was 6°.

The variability in angular velocity in the variable velocity condition and the variability in axis of rotation in the variable axis condition could not be unrestricted. Otherwise, difficulty in maintaining the identity of points from frame to frame (correspondence matches) and in perceiving smooth motion might have confounded the effects of variability. These two factors were controlled first by limiting the variance of the distribution from which the velocities and axis shifts were sampled and then by imposing a correspondence match criterion and a 2D motion criterion (described below) on each display. The mean axis change and mean velocity change in the variable axis and variable velocity conditions was set equal to the axis and velocity changes between views in the fixed axis and velocity conditions. To induce variability within a restricted range, the axis shifts between views in the variable axis condition and the angular rotations between views in the variable velocity condition were sampled from a distribution consisting of the sum of two bimodal distributions. The means of the two distributions were 2° and 10°, respectively. Each distribution had a standard deviation of 0.71 and modes at ±0.7 standard deviations from its mean. A minimum variance criterion for axis shift or angular rotation was used to control for chance selection of nearly equal axis or rotation values across views in the variable axis or variable velocity conditions.

Displays were used only if the following two restrictions were satisfied: In order to reduce the possibility of false correspondence matches of points across pairs of views, the nearest neighbor to any given point, from one view to the next, had to be the correctly corresponding point. This restriction was not applied to points with opposite depth signs, which would be moving in opposite directions. In order to maintain conditions for "short-range" apparent motion across pairs of views, the distance moved by any given point in the image was not allowed to exceed 15' of visual angle (Braddick, 1974).

2.4 *Apparatus*

The stimuli were presented on a Hewlett-Packard Model 1321B X-Y Display with a P-31 phosphor, under the control of a PDP 11/44 computer. The subject viewed the display through a tube arrangement that limited the field of view to a circular area 7.6° in diam. The maximum projected diameter of each simulated object occupied 840 plotting positions on the CRT and subtended a visual angle of 2.1°. The center to center separation of the objects was 3.8°. The eye to screen distance was 1.71 m. The dot and background brightnesses at the screen were approximately 5 cd/m^2 and 0.002 cd/m^2, respectively. A 0.5 neutral-density filter was inserted in the viewing tube to remove any apparent traces on the CRT.

Three models constructed from metal and plastic were used to instruct the subjects. Each model consisted of four white spheres connected by black rods. Two of the models were identical. The third model differed from the others only in the position of one of its spheres. The subjects responded by pressing one of two switches labeled *same* and *different*, respectively. The responses and response latencies were recorded by the PDP 11/44.

2.5 *Procedure*

Subjects were instructed to make *same* or *different* judgments for pairs of stimuli based upon the following criterion: "The two groups are the same when all of the distances between the dots are the same, regardless of their orientation. The two groups of dots are different when the distances between at least two of the dots are different." The three models were used to demonstrate the judgment criterion.

Subjects who were to receive feedback were told that a single tone would indicate a correct response and that two successive tones would indicate an incorrect response. The room was darkened 2 min before the trials began.

Each subject participated in an initial screening session, 25 experimental sessions, and a follow-up session with debriefing. After the first session, subjects initiated each session individually. Each experimental session consisted of three blocks: a baseline monitoring block and two experimental blocks. The baseline monitoring block consisted of three trials of 30 views at each combination of levels of the points and motion variables, in a random order. These trials were used to assure that the subjects maintained a high level of accuracy when the number of views far exceeds the expected minimum levels. The 30-view trials were also intended to assure that any failure to respond accurately at the minimum view levels was not due to the mental rotation component of the task, which should have been the same on the 30-view trials. Each experimental block consisted of 60 trials, each selected at random from the 60 possible conditions. There was a 2 min rest period between each block of trials.

3. Results

The subject's task may be interpreted as determining whether or not there was a difference between the 3D structures represented by the standard and comparison stimuli. A signal detection paradigm (Green & Swets, 1966) was used to analyze the results, with the *different* trials serving as signal trials. A d' measure wascomputed for each subject and stimulus condition, using the proportion of *different* responses on *different* trials as the hit rate and the proportion of *different* responses on *same* trials as the false alarm rate. Each d' was based on 50 trials, approximately half of which were signal (*different*) trials (see Appendix A for d' scores).

An analysis of variance revealed significant main effects for number of points, $F(3, 15) = 14.99$, $p < .01$, and number of distinct views, $F(4, 20) = 28.91$, $p < .01$. As shown in Figure 2, d' decreased with increasing numbers of points. This is opposite to the result that would be expected if the theoretical analyses in Table 1 were used to recover the 3D structures from the 2D images. This unexpected result is probably due to differences between the idealized process of recovery of structure and the requirements of the comparison task used in the present experiment. These differences will be discussed below.

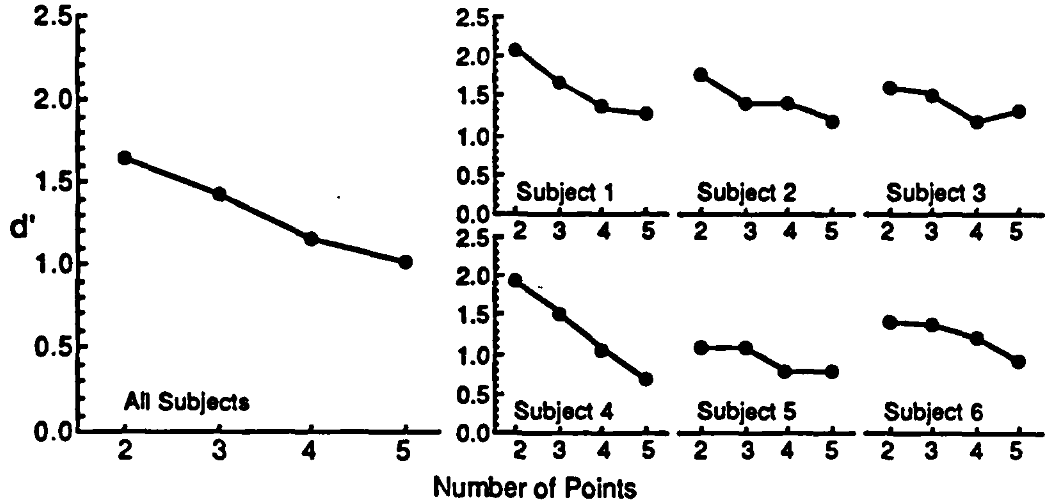

Figure 2. Effect of number of points on d' for 2-6 views. (Data points for individual subjects are based on 750 observations.)

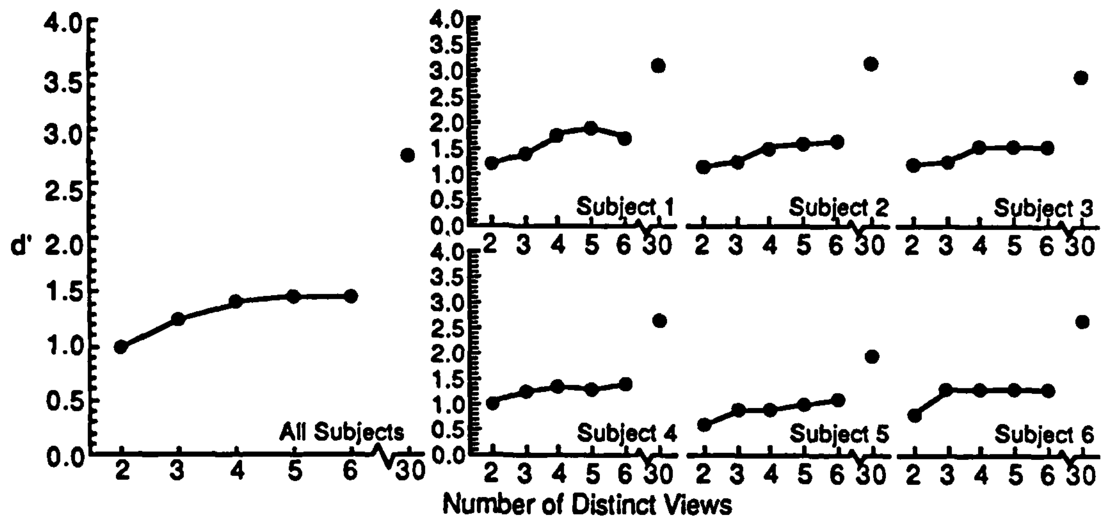

Figure 3. Effect of number of distinct views on d'. (Data points for individual subjects are based on 600 observations.)

As expected, d' increased with increasing number of views (see Figure 3). The main effect of motion constraint was not significant, $F(2, 10) = 3.12$, $p > .05$, but there was a significant interaction of motion constraint with points, $F(6, 30) = 4.74$, $p < .01$, and a second order interaction of motion constraint with points and views, $F(24, 120) = 3.51$, $p < .01$. The motion/points interaction, illustrated in Figure 4, shows a sharper drop in accuracy after three points in the variable axis condition than in the other two motion conditions. Separate inspection of the proportions of correct responses for the *same* and *different* trials (Figure 5) indicates that this interaction, as well as the main effect of points, was primarily due

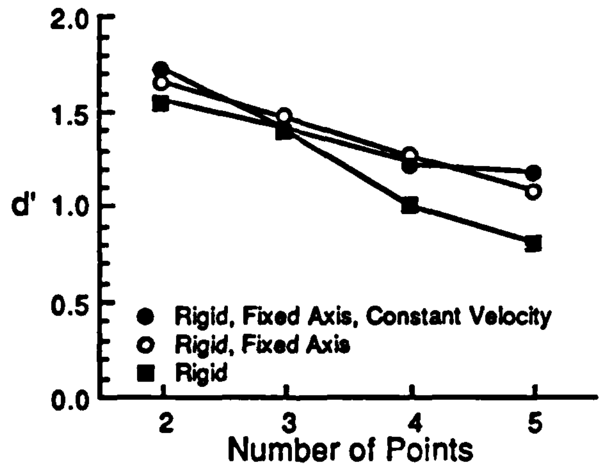

Figure 4. Interaction of number of points with motion condition for 2-6 views.

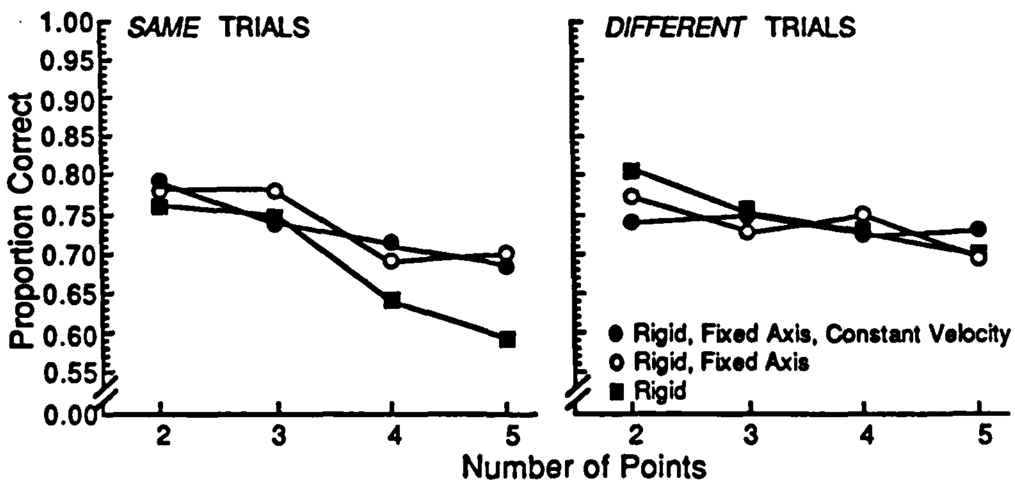

Figure 5. Interaction of number of points with motion condition for *same* and *different* trials.

to the *same* trials. There was also a significant interaction of number of points with number of distinct views, $F(12, 60) = 2.78$, $p < .01$ (see Figure 6). The magnitudes of the significant effects, ω^2, for points, views, points by views by motion constraint, points by views, and points by motion constraint, were .172, .099, .072, .025, and .018 respectively. The main effects of motion constraint, $F(2, 10) = 3.12$, and the interaction of motion constraint with views, $F(8, 40) = 1.31$, were not significant, $p > .05$.

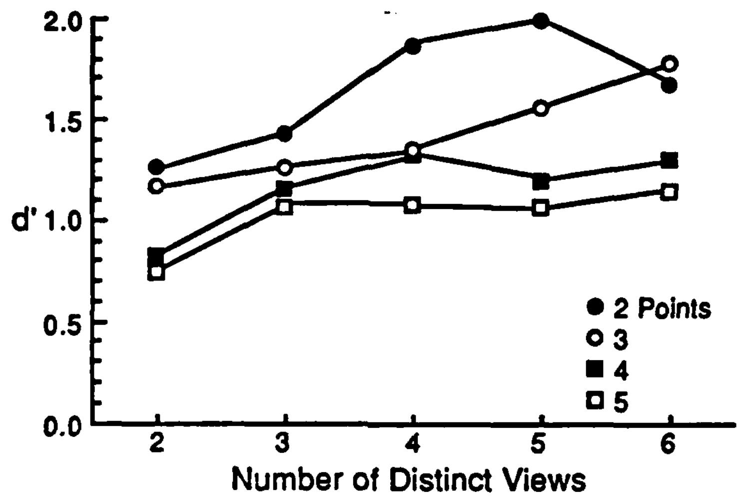

Figure 6. Interaction of number of points with number of distinct views.

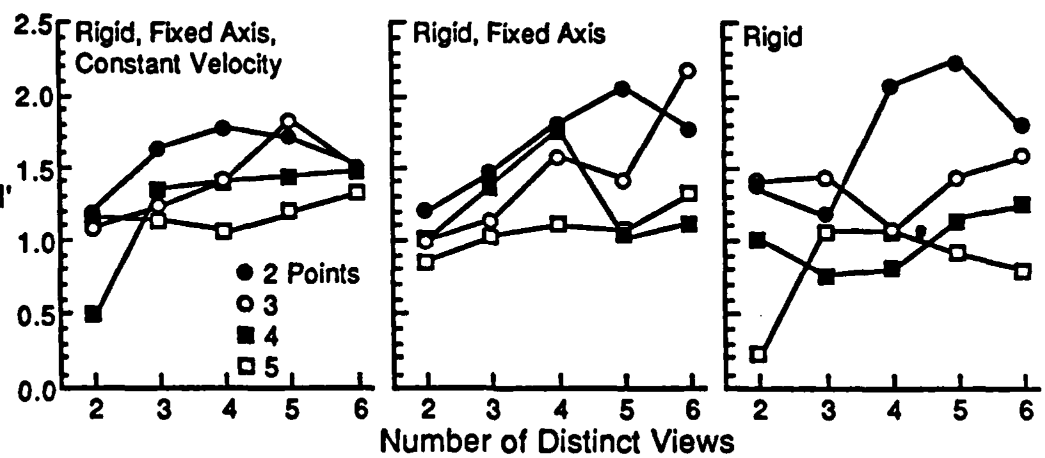

Figure 7. Interaction of number of points, number of distinct views, and motion condition.

A second order interaction would be expected if different combinations of points and views are required to recover structure under different motion constraints. Although a significant second order interaction occurred (Figure 7), there is no indication that this interaction was related to the critical combinations of points, views, and motion constraints listed in Table 1. The significance of the d' scores was calculated for each subject and points/views/motion combination using Marascuilo's one signal significance test (1970, pp. 238-240). In general, d' decreased as expected when the number of views dropped below the levels required in the critical combinations, but d' did not drop to chance as soon as the number of views dropped below the number theoretically required to recover structure, for a given combination of points and motion constraints. On the other hand, d' generally increased as the number of points was reduced which, as noted above, is not consistent with the theoretical anlayses of recovery of structure. Of the 360 d' scores, 351 were significantly different from zero, $p < .05$ (see also Appendix A). For three subjects, d' dropped to chance levels when the fixed axis constraint was removed for five points and two distinct views. (This is not one of the critical combinations, however.) There was no systematic pattern to the remaining

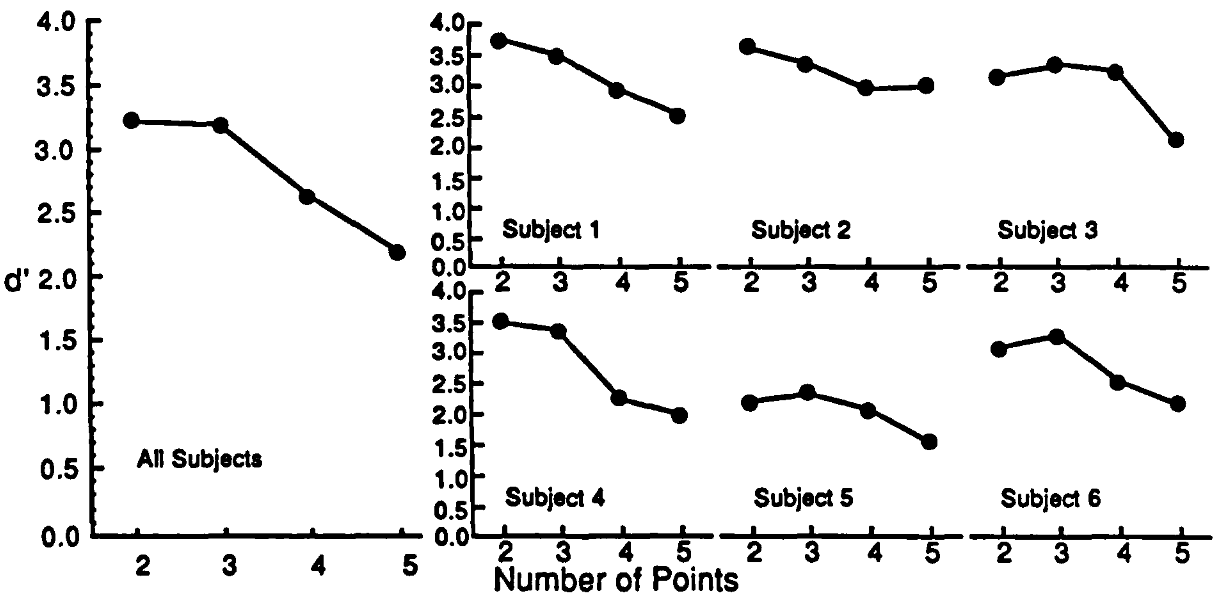

Figure 8. Effect of number of points on d' for 30 views. (Data points for individual subjects are based on 1125 observations).

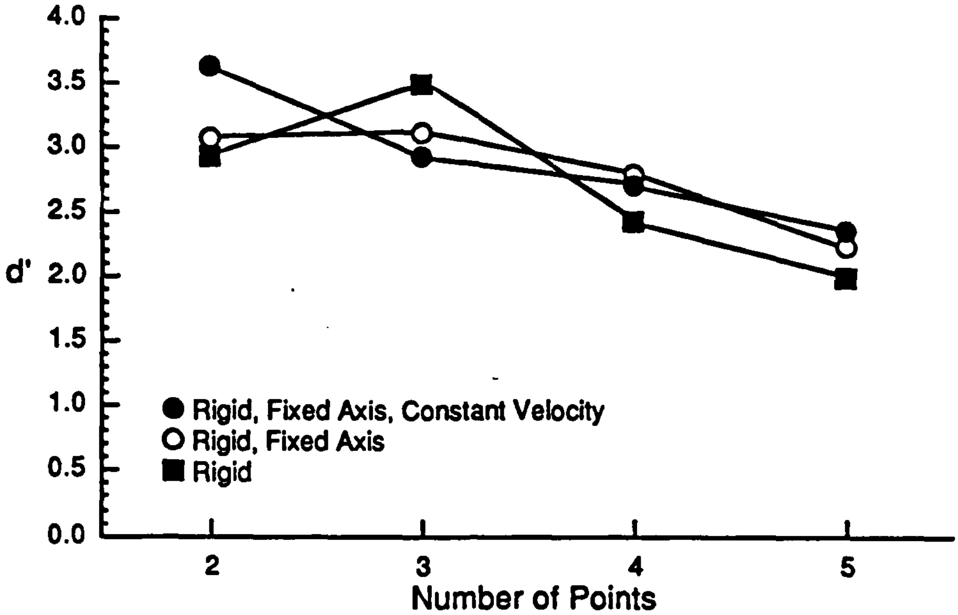

Figure 9. Interaction of number of points with motion condition for 30 views.

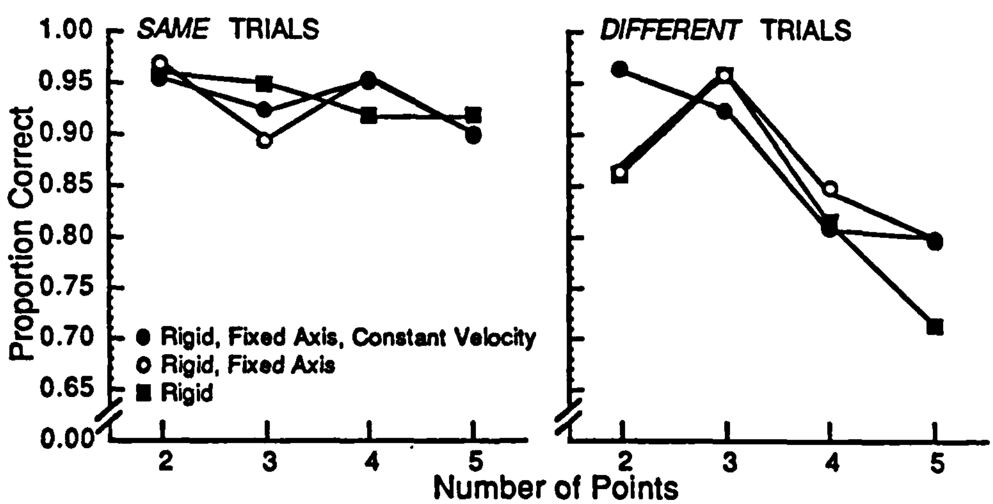

Figure 10. Interaction of number of points with motion condition for *same* and *different*, 30 view trials.

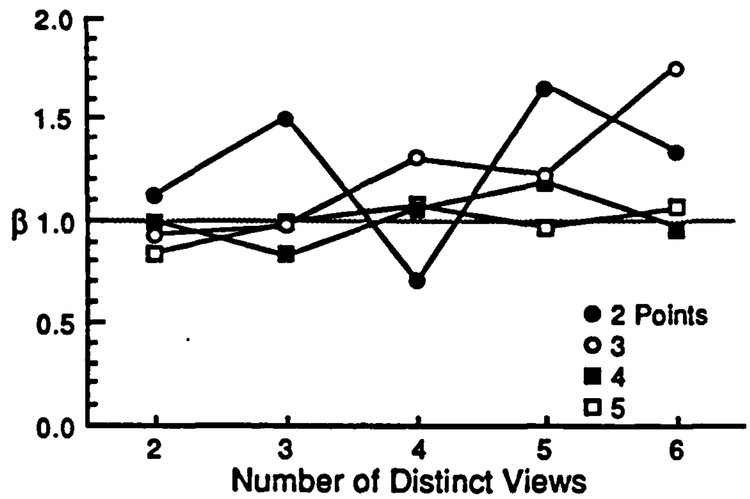

Figure 11. Interaction of number of points with number of distinct views for β.

nonsignificant d's, except that half of these were due to one of the subjects (No. 5). Overall, the critical combinations presented in Table 1 do not seem to be relevant to human performance in the comparison task used in this experiment.

Significant d's were found for all subjects in the two-point/two-view conditions, for all three motion conditions. This is a remarkable finding as that combination of points and views falls below the minimum required to recover 3D structure under the constraints used in existing structure-from-motion proofs. The possibility that subjects exploited additional constraints in the present experiment will be discussed below.

A separate analysis was conducted for the d' measure for 30-view trials. The main effect of number of points was significant, $F(3, 15) = 19.92$, $p < .01$, $\omega^2 = .311$ (see Figure 8). The d' values for two, three, four, and five points were 3.21, 3.18, 2.64, and 2.20, respectively. The interaction of motion condition with points was also significant, $F(6, 30) = 2.81$, $p < .05$, $\omega^2 = .052$ (see Figure 9). This interaction was generally similar to that found for the 2-6 view conditions, although, as seen in Figure 10, the reduction in accuracy with increasing number of points seems to be due primarily to *different* trials. The main effect of motion condition was not significant, $F(2, 10) < 1$.

An analysis of variance of the subjects' response biases, β, for all subjects and conditions revealed significant effects only for number of distinct views, $F(4, 20) = 4.25$, $p < .05$, and for the interaction of number of points with number of views, $F(12, 60) = 2.33$, $p < .05$. (The ω^2 values for these two effects were .019 and .037, respectively.) As the number of views increased, subjects' bias changed from no bias to a *different* response bias. This bias was most noticeable for five or six views of two or three points (see Figure 11).

An analysis of variance was conducted for the mean response latencies (see Appendix B) for the 2-6 view conditions. An additional independent variable, trial type (*same* or *different*) was used in this analysis. (This variable does not appear in the d' analysis because both types of trials were used in computing d'.) The number of points accounted for most of the variance, $F(3, 15) = 47.1$, $p < .01$, $\omega^2 = .657$. The mean latencies for 2-5 point displays were 3.60, 4.86, 8.08, and 10.58 sec, respectively. There were no other significant main effects. There were four significant interactions: points with motion, $F(6, 30) = 3.24$, $p < .05$; points with views with motion, $F(24, 120) = 2.42$, $p < .01$; trial type with views, $F(4, 20) = 7.62$, $p < .01$; and points with type with views, $F(12, 60) = 2.29$, $p < .05$. All of these interactions were relatively small; the ω^2 values for the four interactions were .015, .032, .052, and .016, respectively. If subjects compared all interpoint distances across displays on *same* trials, but responded as soon as one nonmatching distance was found on *different* trials, there should have been an interaction of number of points with type of trial. This interaction was not significant, $F(3, 15) = 1.14$, $\omega^2 = .0013$. A comparison of these values to the large main effect found for number of points indicates that this type of differential processing of *same* and *different* trials did not occur.

In response to debriefing questions, all but one of the subjects reported seeing the simulated objects as 3D on 80-100% of the trials. One subject, No. 5, reported only 2D interpretations for the first 12 sessions and 3D interpretations on 90% of the trials in the latter 13 sessions. Figures 1 and 2 show the d's for this subject to be consistently below those of the other five subjects.

4. Discussion

Theoretical analyses of the recovery of 3D structure from 2D images have shown that three views of four points are sufficient to recover structure under a rigidity constraint (Ullman, 1979), with fewer points required as additional constraints and/or views are added (Hoffman & Bennett, 1985; 1986). As these proofs assume infinitely fine resolution and the absence of noise, one might expect poorer performance by the human observer. More views should be required if an incremental rigidity scheme (Ullman, 1984) were used to overcome the effects of noise. Human performance in our experiment might be expected to be degraded further because of the interposition of a task possibly requiring mental rotation between the recovery of structure and the behavioral response. For these reasons, our finding that subjects could make accurate psychophysical judgments with fewer points and distinct views than expected on the basis of theoretical analyses is especially surprising. It should of course be emphasized that the theoretical analyses are concerned with recovery of the 3D coordinates of points in an arbitrarily-scaled space, up to a reflection about the image plane, whereas our subjects were comparing pairs of structures. The implications of these differences between the theoretical concept of recovery of structure and the requirements of our behavioral task will be discussed below. Still, it is remarkable that any task in which subjects were required to respond to 3D relationships based on information in dynamic monocular displays could be performed on the basis of as few as two points and two distinct views.

Subjects should not have been able to perform significantly above chance with two points and two distinct views if the only constraint incorporated in their processing of the image data was rigidity, or even if fixed axis and constant angular velocity constraints were applied. This suggests that subjects exploited additional constraints that are not included in the theoretical analyses. In a mathematical analysis, it is possible to specify a limited set of applicable constraints, such as rigidity, fixed axis of rotation, and constant angular velocity. In presenting displays to human subjects, however, additional constraints may become applicable. First, the subjects may exploit the constraint of a constant relationship, across displays, between the distances in the projection and the 3D distances that are represented. Scale is undetermined in the mathematical analyses, and the subjects indeed may impose an arbitrary scale in recovering the 3D structures of the displays. It seems unlikely that different scales would be imposed on different displays, however, especially for displays within the same pair. The assumption that the scale is the same for both displays in a pair is essential to accurate responding to the two-point displays. Indeed, we could not have scored a *same* or *different* response to a pair of two-point displays as correct or incorrect without assuming equal scales.

Second, in the mathematical analyses discussed above, rigid translations of the configuration of points do not provide information about the structure. It is assumed, for example, that the subject fixates one point. (Eye movements required to maintain this fixation are not considered as sources of information for use in recovering the structure.) In our experiment, the subjects may have used the lack of translation of the center of rotation as an additional constraint. (To match the theoretical analyses, it would be necessary to link the image to movements of the subject's eye so that one point was always fixated, but no eye movements were required to maintain the fixation.)

Third, in the variable velocity and variable axis conditions, subjects may have exploited the constraint that these variations occurred within a limited range. These limitations were necessary to meet other requirements of structure-from-motion analyses. These analyses are applicable only if the correspondence of points from frame to frame is correctly resolved. If a nearest neighbor criterion is postulated for correspondence matches, displayed points should not be moved to positions that bring them closer to the previous position of another point than to their own previous position. In general, this restricts the amount of rotation to be displayed between views. We limited our displays to distances within the short-range motion process (Braddick, 1974), further restricting the amount of rotation. These restrictions may have reduced potential differences between the three motion conditions.

Our finding of above chance performance with as few as two points and two views was not the only unexpected result. Recovery of structure, for small numbers of points and views, might be expected to be more accurate as the number of points is increased. We found, on the contrary, that accuracy decreased with increasing numbers of points. This result is probably another reflection of the differences between the theoretical operation of recovering structure and the requirements of a behavioral task. Increasing the number of points in an object may enable a subject to recover the 3D structure with fewer views or motion constraints. On the other hand, increasing the number of points increases the complexity of the structure, in terms of the number of interpoint distances. For this reason, any task used to ascertain that the relative depth coordinates of all points in the structure have been correctly recovered is likely to be more difficult for greater numbers of points. This appears to have been the case for the comparison task in the present experiment. This conclusion is supported by the latency results: Response time increased as the number of points in the structures increased.

We found that the decrease in accuracy with increasing numbers of points occurred primarily in the *same* trials. This finding is consistent with an explanation of the relationship between accuracy and the number of points based on the subject's attempting to compare interpoint distances. On a *different* trial, the subject would need only to be certain that at least one interpoint distance was not the same in both stimuli in a pair. On *same* trials, the subject would have to keep track of all of the interpoint distances. Failure to detect any mismatch in distances could lead to an incorrect response. There was no corresponding difference in latency for *same* and *different* trials, however, suggesting that the number of comparisons required was not different for the two types of trials. There is another possible reason for higher accuracy on the *different* trials. To verify that two structures are the same, each must be recovered uniquely. To determine that two structures are different, it is only necessary to identify a set of possible structures for each display, and to determine that these two sets do not overlap.

The drop in accuracy with increasing numbers of points on *same* trials was especially marked after three points for the variable axis condition. Verbal reports indicated that the subjects attempted to organize the display into subunits of no more than three points. A four-point display might be perceived as a triangle and a dot; a five point display as a triangle and a rod. Our hypothesis is that it was more difficult to maintain a perception of rigid relationships among subunits for the variable axis displays. The use of triangular subunits by subjects in these judgments, and the importance of triangles in the analysis of optic flow (Koenderink & van Doorn, 1986), may be more than coincidental.

These suggestions of possible grouping effects indicates that organization of feature points into subgroups should be examined as a potentially important component of the recovery of 3D structure from dynamic 2D images. Some principles for grouping based on orthographic projections of rotation in depth have been reported by Gillam (1976). In studies of the recovery of structure from motion, it would be important to determine whether grouping was based on 3D or on 2D relationships. This might indicate whether grouping or recovery of structure occurred first, or perhaps would show that the two processes occur in parallel.

In summary, we have used an indirect method to test theories about the recovery of structure from motion that are not directly testable. We found that human observers could perform a structure comparison task at better than chance levels with fewer points and views than should have been necessary for the recovery of separate structures. This implies that constraints other than those considered in these theories were exploited by the subjects. Some of these possible constraints might be incorporated into theoretical analyses of the recovery of structure from minimum points and views: (a) Even when scale is undetermined, the scale used to recover structure may not be arbitrary, but may depend on assumptions of constant scale or an assumption of a specific scale. These assumptions correspond to the equidistance tendency and the specific distance tendency (Gogel, 1973).

(b) Information that specifies a fixed center of rotation, as might occur when rotation is induced by observer head movements, may be used in the recovery of structure. (c) Even when precise constant velocity and fixed axis assumptions are not applicable, most motions occur within a limited range of velocity change and axis change--that is, velocity and axis of rotation usually changes smoothly.

There remains a general lack of direct tests of mathematical theories of the recovery of structure from motion. To achieve such tests, it may be necessary to elaborate existing theories to include direct implications about human competency in tasks that are subject to psychophysical testing. It may also be necessary to develop psychophysical techniques that are more suitable for the study of dynamic information for depth perception than current techniques, which emphasize detection of minimal differences. The issue of interest in this research is often how a particular flow pattern leads to a particular perception, rather than whether an experienced observer can discriminate minimum differences in optic flow. As such developments in mathematical analysis and in psychophysics proceed, it should become possible to combine mathematical and psychophysical approaches to the study of more complex patterns of optic flow (Koenderink, 1986).

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Appendix A

Response data, d', by subject, number of distinct views, number of points, and motion condition (C = constant angular velocity, fixed axis, rigid; F = fixed axis, rigid; R = rigid). (†Nonsignificant scores, $p > .05$.)

	Number of points											
	2			3			4			5		
Motion condition: (v = Number of distinct views)												
v :	C	F	R	C	F	R	C	F	R	C	F	R
Subject 1												
30 :	4.2879	3.3400	3.5438	3.0206	3.6092	3.7930	3.1758	3.2825	2.3450	2.8016	2.3500	2.3981
2 :	1.8064	1.1316	1.4394	0.8851	1.4202	1.5442	0.5206	0.9337	1.3723	1.5275	1.3529	0.2777
3 :	1.5207	1.5488	1.6698	1.4109	1.4358	1.4700	1.4275	1.1718	0.9714	1.0458	1.7005	1.5174
4 :	1.9361	2.5642	3.1189	1.7792	1.8159	1.1769	1.7648	2.2563	0.7618	1.5208	1.6836	0.9853
5 :	2.1012	3.5695	2.3932	1.9361	1.6690	1.9530	1.8890	1.3865	1.7017	1.2897	1.1559	1.5566
6 :	1.9664	2.5932	2.1580	2.2279	2.2546	1.8853	1.4510	1.5397	1.2791	1.5442	1.5208	0.5348
Subject 2												
30 :	3.2255	3.5797	3.9604	3.2272	3.2037	3.5462	3.4300	2.9751	2.3450	3.7930	3.0287	2.1915
2 :	1.2975	1.1765	1.3660	1.6933	0.6074	1.6949	0.8731	1.3633	1.1822	0.8819	1.1824	0.1794†
3 :	1.8026	1.4109	0.8190	1.6531	0.6066	0.9330	1.1670	1.3697	0.4853	2.4040	1.0887	1.1636
4 :	2.3042	1.8064	2.1275	0.8685	1.5256	1.1769	2.2199	1.8259	1.2678	0.6556	1.0951	1.3756
5 :	1.5442	2.6124	2.3736	2.5220	1.6725	1.2047	1.7017	1.2323	1.4746	1.0458	1.3697	0.8099
6 :	2.1967	2.0172	1.6937	1.4079	2.0129	1.3833	1.8872	1.1805	1.5832	1.4700	1.4202	1.8002
Subject 3												
30 :	3.9727	3.1798	2.3303	3.5485	2.4694	3.9962	3.4300	2.9751	3.2199	2.1163	1.8944	2.1417
2 :	1.2210	1.7284	1.3698	1.2700	1.4500	1.2897	0.4591	1.3389	1.0197	1.6257	0.5885	0.5470
3 :	1.5299	1.1670	0.8883	0.7649	1.0406	1.5828	0.9090	1.7330	0.7460	0.9468	1.7717	2.0881
4 :	1.9343	1.9566	1.3690	1.7041	2.0103	1.2080	1.7648	2.1430	0.1919†	1.7284	1.2892	0.9853
5 :	1.7041	2.2138	2.0045	2.2699	1.2791	1.3529	1.4746	0.7516	0.9984	1.5488	1.5460	1.0569
6 :	1.1326	2.0172	1.6631	1.7039	2.1580	1.6631	1.5442	1.2978	1.0386	2.3944	1.1765	0.3769
Subject 4												
30 :	3.8637	3.5797	3.0506	2.6512	4.1618	3.2195	1.8599	2.6150	2.1911	2.3077	2.1292	1.4175
2 :	1.3661	1.7107	1.7443	0.9974	1.1765	1.1769	0.7021	1.0207	1.2791	1.0197	0.5938	0.0245†
3 :	2.2500	1.8872	1.3865	1.3003	1.5460	1.6949	2.1200	1.2863	0.1013†	0.8209	0.6043	0.6060
4 :	2.3233	2.1239	2.4831	1.2897	1.6725	1.2610	0.7258	1.2863	1.1326	1.0702	0.6560	0.6227
5 :	1.8373	1.9386	2.6133	1.5576	1.1472	1.3661	1.1140	1.3254	1.1140	1.0458	0.9425	0.2761
6 :	2.0129	1.4246	2.1580	1.3697	3.2721	2.1271	1.1636	0.5779	1.3139	0.4303	0.9819	0.5891
Subject 5												
30 :	2.4866	2.1150	2.0033	1.9972	2.3159	2.6353	1.8153	2.6573	1.6666	1.2421	1.5855	1.7248
2 :	0.6500	0.7324	1.0019	0.9974	0.4226	1.1769	0.1272†	0.1886†	0.3328	1.0920	0.7486	-0.1338†
3 :	0.4329	1.0508	1.3254	0.8411	0.8195	1.4700	1.2474	1.0569	1.1714	0.8411	0.1575†	0.4048
4 :	0.9520	0.4972	1.9631	0.9441	0.9146	0.6118	0.6081	1.1714	0.4349	0.6785	1.0951	1.1288
5 :	1.6531	1.0216	1.5698	1.3254	1.3963	1.4920	0.9279	0.5266	0.4098	1.0064	0.4803	0.8109
6 :	0.4591	1.0951	1.7089	0.9215	1.9049	0.8980	1.3003	0.9214	1.2772	0.8209	1.7029	0.7492
Subject 6												
30 :	3.8637	2.6109	2.7389	3.0168	2.9841	3.8587	2.5156	2.3052	2.7563	1.9066	2.3500	2.1417
2 :	0.9550	0.6858	1.2072	0.7043	0.9126	1.5689	0.0963†	1.1636	0.9060	0.8731	0.7486	0.4377
3 :	2.2500	1.7659	0.9264	1.4275	1.3697	1.5689	1.2897	1.6854	1.1757	0.8209	0.9279	0.7127
4 :	1.2070	1.9050	1.4225	1.8859	1.5801	1.0549	1.4025	1.8896	1.0258	0.7324	0.8573	1.3571
5 :	1.4092	1.0197	2.5642	1.3306	1.3566	1.3661	1.5608	0.9233	1.2791	1.1769	0.9425	0.9425
6 :	1.3225	1.4624	1.2028	1.4358	1.5627	1.6642	1.5464	1.2791	1.0386	1.2897	1.1459	0.7492

Appendix B

Response latencies in seconds, by subject, trial type, number of distinct views, number of points, and motion condition (C = constant angular velocity, fixed axis, rigid; F = fixed axis, rigid; R = rigid).

	Number of points											
	2			3			4			5		
Motion condition: (v = Number of distinct views)												
v :	C	F	R	C	F	R	C	F	R	C	F	R
Subject 1												
Same trials												
30 :	4.78	4.59	5.53	5.87	5.89	6.23	7.80	6.70	7.95	9.24	10.17	12.36
2 :	3.60	4.44	4.95	6.04	5.04	5.12	6.46	8.12	7.64	12.42	9.99	10.77
3 :	3.75	3.95	3.93	5.76	6.35	6.89	10.96	9.25	8.61	11.91	13.68	11.52
4 :	4.87	4.35	4.31	6.00	6.30	4.03	8.76	9.51	7.20	9.59	9.56	12.02
5 :	3.89	4.40	4.40	4.74	6.36	5.68	9.36	8.82	10.83	12.96	12.88	11.17
6 :	4.45	4.07	3.81	4.49	4.76	4.40	9.30	8.91	7.43	13.17	12.41	11.32
Different trials												
30 :	5.38	4.26	3.67	5.16	6.16	4.84	9.01	9.23	7.34	12.46	11.09	11.08
2 :	4.14	4.70	5.03	5.45	6.60	3.97	9.27	7.20	8.40	10.40	10.52	11.52
3 :	4.51	5.74	3.10	5.95	5.48	5.93	7.17	9.39	9.48	12.70	9.53	10.96
4 :	3.97	3.98	4.80	4.97	5.99	5.08	10.46	8.38	10.59	15.09	10.75	11.61
5 :	5.11	4.54	4.75	5.72	5.21	5.45	8.42	9.15	9.26	8.64	11.68	10.90
6 :	4.58	3.64	5.01	5.05	4.70	6.70	8.70	6.60	9.88	14.12	9.18	10.66
Subject 2												
Same trials												
30 :	3.54	2.51	3.44	3.85	5.48	4.36	4.51	4.30	8.39	6.31	7.22	13.31
2 :	3.43	4.07	3.43	3.29	3.81	4.56	7.36	7.09	7.01	9.59	9.60	12.97
3 :	2.16	4.28	3.60	6.19	4.50	6.30	8.40	9.73	9.98	13.88	14.13	15.93
4 :	4.61	3.26	3.79	5.69	7.79	7.12	9.98	13.47	11.90	17.83	16.61	19.27
5 :	4.25	4.63	5.37	6.95	9.17	6.66	9.03	9.36	11.13	14.19	16.91	15.01
6 :	3.85	4.00	4.05	5.92	5.83	5.83	12.13	10.17	10.74	15.24	14.80	15.16
Different trials												
30 :	3.40	3.25	4.53	5.56	4.49	6.11	5.33	9.35	11.31	11.89	14.56	14.38
2 :	4.95	4.65	3.72	6.52	7.31	6.31	8.70	10.47	9.56	17.99	15.04	17.59
3 :	4.86	3.74	2.49	5.02	5.74	5.16	7.24	9.06	9.39	11.86	14.97	12.06
4 :	4.60	3.48	5.03	9.18	5.63	7.32	10.61	11.35	12.99	21.57	17.45	18.71
5 :	3.12	4.61	4.93	5.11	4.39	5.69	6.38	11.37	11.05	12.58	12.85	13.07
6 :	4.02	4.57	4.30	5.91	5.92	8.07	9.83	10.24	9.14	15.02	17.69	16.22
Subject 3												
Same trials												
30 :	5.87	4.17	5.05	6.14	6.72	6.05	7.64	7.70	8.98	10.80	8.80	9.38
2 :	4.58	4.50	4.98	5.88	6.46	6.44	10.61	11.96	10.70	14.75	14.94	14.83
3 :	3.40	3.48	3.49	5.58	6.51	6.02	11.47	7.59	7.68	12.21	11.62	10.44
4 :	2.91	3.44	3.98	3.67	4.77	6.03	6.83	7.98	8.25	10.87	8.11	11.56
5 :	3.34	4.13	3.67	5.27	3.68	5.12	9.86	10.25	9.63	13.33	10.72	11.22
6 :	3.41	2.91	3.46	3.92	4.19	3.90	6.99	9.27	8.71	10.69	8.87	11.94
Different trials												
30 :	3.26	3.82	3.95	5.20	3.61	5.41	7.29	9.09	8.02	9.27	13.30	11.30
2 :	3.28	2.68	3.57	5.82	4.85	4.29	7.88	9.04	7.15	12.14	13.16	9.84
3 :	3.01	3.50	2.98	4.62	4.44	4.15	7.23	9.14	9.45	11.11	7.88	10.43

4 :	2.85	2.95	2.69	4.97	5.15	4.89	9.10	9.38	9.91	12.04	12.17	11.42
5 :	3.91	3.52	3.97	6.02	4.14	5.02	7.60	8.44	9.09	7.58	8.71	12.76
6 :	4.87	3.30	4.05	4.54	4.47	5.49	7.60	9.36	10.07	12.27	11.87	12.60
Same trials												
Subject 4												
30 :	3.16	3.25	3.47	4.71	4.16	3.82	7.17	5.40	6.53	6.48	6.68	7.70
2 :	3.15	3.41	3.76	4.31	4.15	4.36	6.22	6.18	7.53	9.34	8.42	8.33
3 :	3.11	3.68	4.03	3.54	4.58	3.91	6.98	8.88	7.05	13.04	8.17	9.92
4 :	4.06	2.95	3.14	3.30	5.14	3.73	8.43	8.25	6.59	10.93	9.45	8.85
5 :	3.49	4.05	3.94	4.78	7.07	6.90	7.17	8.39	6.96	10.33	11.86	9.71
6 :	3.16	3.35	2.68	3.83	4.14	5.60	6.67	7.49	7.66	9.59	8.49	9.89
Different trials												
30 :	2.78	3.29	3.56	5.09	4.84	4.88	8.11	8.64	8.11	8.53	8.72	10.59
2 :	2.85	2.41	2.81	4.73	3.28	4.39	6.72	8.77	6.50	8.97	11.26	10.12
3 :	3.22	4.04	2.58	4.44	4.22	4.78	5.89	7.76	7.07	11.23	9.51	10.30
4 :	3.01	3.25	2.88	4.22	4.40	3.66	9.20	10.59	10.54	11.45	8.77	8.79
5 :	3.34	2.50	3.64	6.17	4.47	4.78	7.65	7.55	9.33	9.67	10.65	9.05
6 :	3.42	3.35	2.20	3.98	4.13	5.07	5.75	8.82	9.38	10.75	10.20	11.29
Subject 5												
Same trials												
30 :	6.60	5.05	6.67	8.12	6.76	6.33	8.10	6.92	8.03	9.80	8.68	8.90
2 :	9.79	7.24	7.33	10.26	8.22	8.32	12.95	9.77	12.12	11.76	13.70	10.90
3 :	2.59	2.96	2.05	3.57	4.39	3.75	6.74	6.21	6.09	5.85	7.95	9.52
4 :	2.73	2.85	1.56	2.80	3.71	3.16	6.22	6.77	5.54	7.79	6.10	5.71
5 :	2.79	2.70	3.29	3.23	5.10	3.16	7.33	7.54	5.70	7.14	8.00	7.43
6 :	3.41	2.00	1.91	3.22	3.32	3.73	5.14	5.13	6.42	6.38	6.40	7.13
Different trials												
30 :	2.54	2.99	2.46	4.10	3.38	3.65	7.90	6.59	4.74	7.71	6.48	7.46
2 :	2.12	2.19	2.39	3.63	2.41	2.96	5.48	4.58	6.63	5.91	7.99	5.75
3 :	2.68	3.12	2.43	3.23	3.73	3.53	4.83	5.74	6.76	8.16	7.30	5.57
4 :	2.27	2.03	2.16	4.44	3.84	2.96	6.46	7.52	5.35	7.70	6.18	5.44
5 :	3.56	3.42	2.81	3.77	3.96	4.56	7.37	5.77	6.37	6.70	9.20	6.71
6 :	5.09	1.83	2.71	4.15	3.60	4.55	6.60	6.61	8.94	7.78	7.29	7.77
Subject 6												
Same trials												
30 :	4.78	4.96	4.37	6.64	5.46	6.27	8.24	6.25	6.80	9.66	8.26	11.51
2 :	4.98	4.35	6.27	5.69	5.13	4.92	6.83	6.46	8.71	9.77	8.46	12.08
3 :	3.02	4.10	3.76	3.66	4.74	4.82	8.38	7.21	6.52	9.39	8.58	7.70
4 :	4.23	3.09	3.74	4.88	4.15	4.08	5.10	5.35	7.22	9.60	6.28	8.00
5 :	3.88	3.45	4.62	3.54	5.04	4.42	7.33	4.98	6.56	8.48	8.33	7.65
6 :	4.13	3.43	3.82	3.60	4.69	3.57	6.23	8.18	5.88	9.07	8.23	9.36
Different trials												
30 :	2.93	3.75	2.94	4.61	4.55	4.40	6.50	6.54	7.84	10.09	7.39	9.42
2 :	4.81	4.37	3.45	4.22	4.47	3.88	6.34	5.79	6.30	8.10	7.48	7.90
3 :	3.16	3.47	3.90	4.47	4.16	4.08	6.79	7.23	7.61	8.64	7.93	6.89
4 :	4.16	3.95	3.49	5.52	4.57	4.57	6.76	7.57	7.96	8.11	8.00	10.00
5 :	4.31	2.96	3.98	3.67	4.58	3.64	6.96	5.91	6.08	7.98	7.84	8.47
6 :	4.22	4.74	3.56	5.08	5.18	5.04	6.24	6.86	6.82	9.11	7.17	7.49

August 19, 1986

Addendum

We are currently studying performance on the structure comparison task with a single (stationary) view of each simulated object. Preliminary results indicate that above chance performance may be possible in this condition. These new results may alter the interpretations presented in this report.

Erratum

P. 20. The page numbers for Hoffman and Bennett, 1986, should be 71-83.

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